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A matheuristic for the liner shipping network design problem with transit time restrictions

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Abstract. We present a mathematical model for the liner shipping network design problem with transit time restrictions on the cargo flow. We extend an existing matheuristic for the liner shipping network design problem to consider transit time restrictions. The matheuristic is an improvement heuristic, where an integer program is solved iteratively as a move operator in a large-scale neighborhood search. To assess the effects of insertions/removals of port calls, flow and revenue changes are estimated for relevant commodities along with an estimation of the change in the vessel cost. Computational results on the benchmark suite *LINER-LIB* are reported, showing profitable networks for most instances. We provide insights on causes for rejecting demand and the average speed per vessel class in the solutions obtained.

Keywords: Liner shipping; network design; transit time.

1 Introduction

The *liner shipping network design problem with transit time restrictions*, LSNDP-TT, is a core planning problem facing container carriers. The problem is to design a set of cyclic routes for container vessels to provide transport for commodities in an origin-destination (OD) matrix respecting transit time restrictions of each individual commodity. The objective of the problem is to maximize the profit of the liner shipping company through the revenues gained from container transport taking into account the fixed cost of deploying vessels and the variable cost related to the operation of the routes and the handling cost of cargo transport. As a consequence of maximizing profits the liner shipping network design problem generally allows rejection of some commodities if deemed unprofitable.

Recent literature on liner shipping network design allows arbitrary transit times for all commodities (Brouer et al., 2014b; Liu et al., 2014; Wang and Meng, 2014; Mulder and Dekker, 2014; Brouer et al., 2014a; Plum et al., 2014; Reinhardt and Pisinger, 2012; Gelareh et al., 2010; Agarwal and Ergun, 2008) although it is generally acknowledged that transit times are decisive for the competitiveness of the network design. This means that from the customer perspective liner shipping network design has multiple objectives as the customers prefer minimal transit times along with low freight rates. However, providing low freight rates by minimizing the cost of the network is likely to result in prolonged transit times as exemplified in Karsten et al. (2015). Likewise designing a network to minimize transit times is likely to result in a very costly network since speed increases. Initial work to construct a multi-criteria objective function is presented in Alvarez (2012) that considers a bi-linear expression for the inventory cost of the cargo on board vessels, but the level of service calculations are not computationally tractable in the already very complex liner shipping network design models. However, the inventory cost of commodities on board vessels is only indirectly a concern to the carrier, when excessive transit times result in the customers switching to a different carrier. Hence, the carriers concern is to ensure a maximal transit time corresponding to the market level of service. Wang and Meng (2014) introduce deadlines on commodities in a non-linear, nonconvex mixed-integer programming (MIP) formulation of liner shipping network design with transit time restrictions. As a consequence the model does not allow transshipment of cargo, which is another common trait of the liner shipping network design problem.

In this paper we present a capacitated multicommodity network design formulation for the liner shipping network design problem allowing for an arbitrary number of transshipments and enabling restrictions on transit time of individual commodities. We also propose an adaptation of the matheuristic of Brouer et al. (2014b) that considers transshipment times to show that it is tractable to incorporate the transit time restrictions in a heuristic context.

The paper is organized as follows: Section 2 reviews related work to the Liner Shipping Network Design Problem, LSNDP, and related areas in maritime and public transportation. Section 3 introduces our mathematical model. Section 4 expands the IP used as a move operator in Brouer et al. (2014b) to consider transit times and the column generation algorithm used to evaluate the cargo flow considering transit times. Section 5 reports computational results for the benchmark suite *LINER-LIB*. We end the paper by drawing conclusions and discussing extensions in Section 6.

2 Literature Review

Meng et al. (2014) and Christiansen et al. (2013) provide broader reviews of recent research on routing and scheduling problems within liner shipping. Here we review selected works on the LSNDP and the inclusion of transit time considerations.

Brouer et al. (2014a) present a thorough problem description of the LSNDP along with a mathematical model and a benchmark suite of data instances. Incorporating transit times into LSNDP is highlighted as an important area for future research. To accommodate future research needs the benchmark instances contain maximum transit time for all OD pairs and these instances are used for the computational results of this paper. Brouer et al. (2014b) develop a matheuristic for the LSNDP. The matheuristic is an *improvement* heuristic according to the categorization in the survey on matheuristics by Archetti and Speranza (2014) meaning that an integer program is used as a move operator. The present paper extends the method of Brouer et al. (2014b) to include transit times.

Alvarez (2012) presents mathematical expressions for the inventory cost of containers during transport. No computational results are reported as the mathematical expressions are not easily incorporated into existing models of the LSNDP. In Wang et al. (2013) an integer program for deciding minimum cost container paths for a single OD pair respecting transit time and cabotage restrictions is considered. Karsten et al. (2015) present a column generation algorithm for a time constrained multicommodity flow (MCF) problem applied to a liner shipping network. A resource constrained shortest path problem is solved for each origin using a specialized label setting algorithm. Different topologies of graphs for liner shipping networks are presented. Computational results for solving the MCF problem with and without transit times on global-sized liner shipping networks are reported. The solution times for the time constrained MCF problem is comparable to solving the MCF problem without transit time restrictions. The algorithm of Karsten et al. (2015) is used in the matheuristic presented in this paper for evaluating a given network during the search. A liner shipping network design problem considering transit time restrictions is presented in Wang and Meng (2014). The model excludes transshipments between services. The problem is proven to be NP-hard and is formulated as a non-linear, non-convex mixed integer program. A column generation based heuristic is developed and a case study is presented for a network of 12 main ports on the Asia-Europe trade lane with three different vessel classes. The model is suggested as an aid to planners in a liner shipping company and the case study provides high quality network suggestions and important insights to assist the planners. The authors suggest incorporation of transshipments along with transit time restrictions as an area of future research.

In this paper we present a mathematical model considering cargo transit time restrictions and transshipments allowed between services. We develop a heuristic solution method incorporating ideas and methods of several of the works mentioned in this section.

3 Mathematical model for the LSNDP-TT

In the following we introduce the notation used to formulate the LSNDP-TT mathematically. An instance of the LSNDP-TT consists of the following sets:

- P: Set of ports with an associated port call cost c^e_p, (un)load cost c^p_U, c^p_L, transshipment cost c^p_T and berthing time b_p spent on a port call.
- K: Set of demands, where each demand has an origin $O_k \in P$, a destination $D_k \in P$, a quantity, q_k, a revenue per unit, z_k, a reject penalty per unit $\tilde{z}_k$ and a maximal transit time, t_k.

- E: Set of vessel classes with specifications for the weekly charter rate, f_e, capacity U_e, minimum (v^e_{min}) and maximum (v^e_{max}) speed limits in knots per hour, bunker consumption as a function of the speed, g^e_v and bunker consumption per hour, when the vessel is idle at ports h^e. There are N_e vessels available of class $e \in E$. The price for one metric ton of bunker is denoted c_B.
- D: Matrix of the direct distances d^e_{ij} between all pairs of ports $i, j \in P$ and for all vessel classes $e \in E$. The distance may depend on the vessel class draft as the panama canal is draft restricted. Along with d^e_{ij} follows an indication of the cost l^e_{ij} associated with a possible traversal of a canal.

A solution to the LSNDP is a subset of the set of feasible services S. A service consists of a set of ports $P' \subseteq P$, a number of vessels, and an average sailing speed. A service is cyclic but may be non-simple, that is, ports can be visited more than once. In this model we allow a single port to be visited twice, yielding a so-called *butterfly* route. The service time T_s is the time needed to complete the cyclic route. A weekly frequency of port calls is obtained by deploying multiple vessels to a service. Let $e(s) \in E$ be the vessel class assigned to a service s and $n_{e(s)}$ the number of vessels of class $e(s)$ required to maintain a weekly frequency. A round trip may last several weeks but due to the weekly frequency exactly one round trip is performed every week. Let v_s be the service speed in nautical miles per hour.

The mathematical model of the LSNPD-TT relies on a set of service variables and a path flow formulation of the underlying time constrained MCF problem. To describe the service network of the LSNDP-TT, we define F^s to be the port sequence $p^s_1, p^s_2, \ldots, p^s_m$ for the service $s \in S$. Let $|s|$ denote the number of unique ports in a service $s \in S$ and $|F^s| = m$ the number of port calls in s.

Furthermore we define a directed graph, $G(V, A)$, with vertices V and arcs A. $V = V_P \cup V_R$ is the set of vertices, where V_P is the subset of vertices representing the unique ports $p \in P$, and V_R is the subset of service vertices representing all port calls by all services. $V_R = \bigcup_{s \in S} V_{F^s}$ and V_{F^s} is the subset of vertices representing the port calls $p^s_1, p^s_2, \ldots, p^s_m$ of service F^s, $s \in S$. $p(v)$ is a function mapping a vertex $v \in V_R$ (i.e., a port call) to its actual port $p \in P$. The set of arcs in the graph can be divided into (un)load arcs, transshipment arcs, sailing arcs, and forfeited arcs, i.e. $A = A_L \cup A_U \cup A_T \cup A_S \cup A_K$. These sets are formally defined below and we associate with each arc $a \in A$ a cost c_a, traversal time t_a, and capacity C_a.

- $A_L = \{(p, v) \,|\, p \in V_P, v \in V_{F^s}\}$ and $A_U = \{(v, p) \,|\, v \in V_{F^s}, p \in V_P\}$ are respectively the sets of loading/unloading arcs representing a departure/arrival at port p visited in F^s, $c_a = c^p_L$, and $c_a = c^p_U$ is the (un)loading cost for a container at the associated port $p \in V_P$, $t_a = 0$, and C_a is unlimited.
- $A_T = \{(v, u) \,|\, v \in V_{F^s}, u \in V_{F^{s'}}\}$ is the set of transshipment arcs representing a transshipment between services F^s and $F^{s'}$ defined for every pair (v, u) where $p(v) = p(u)$, $c_a = c^p_T$ is the transshipment cost for a container at the associated port $p \in V_P$, t_a is the transshipment time, and C_a is unlimited.

- $A_S = \{(v,u) \,|\, s \in S, v,u \in V_{F^s}, v = p^s_h, u = p^s_{((h+1) \bmod m)}\}$ is the set of sailing arcs representing a sailing between two consecutive port calls v and u in F^s, $c_a = 0$ as sailing costs are directly incurred by the vessels, $t_a = d_{ij}/v_s + b_j$ meaning the time in hours to traverse the edge plus the berthing time at the arriving port for each sailing, and $C_a = U_{e(s)}$.
- $A_K = \{(v,u) \,|\, v,u \in V_P, \exists k \in K : O_k = v \wedge D_k = u\}$ is the set of forfeiting arcs representing a rejection of transporting the cargo k between v and u in P, $c_a = \tilde{z}_k + z_k$ is the penalty associated with rejecting the cargo k, $t_a = t_k$ is the maximum transit time for k, and $C_a = q_k$.

We use the path flow formulation of the time constrained MCF problem as described in Karsten et al. (2015). Let Ω_k be the set of all feasible paths for commodity k including forfeiting the cargo. Let $\Omega(a)$ be the set of all paths using arc $a \in A$. The cost of a path ρ is denoted as c_ρ and it includes the revenue obtained by transporting one unit of commodity k sent along path $\rho \in \Omega_k$. The real variable x_ρ denotes the amount of commodity k sent along the path. Let the weekly cost of a service be $c_s = n_{e(s)} f_{e(s)} + \sum_{(i,j)\in A_S} \left(c_B (h^{e(s)} b_p + g^{e(s)}_{v(s)} d^{e(s)}_{ij}) + c^{e(s)}_j + l^{e(s)}_{ij} \right)$ accounting for fixed cost of deploying the vessel and the variable cost in terms of the fuel and port call cost of one round trip. Define binary service variables y_s indicating the inclusion of service $s \in S$ in the solution.

Then the mathematical model of the LSNDP-TT can be formulated as follows.

$$\min \quad \sum_{s\in S} c_s y_s + \sum_{k\in \mathbf{K}} \sum_{\rho \in \Omega_k} c_\rho x_\rho \tag{1}$$

$$\text{s.t.} \quad \sum_{\rho\in\Omega_k} x_\rho = q_k \qquad k \in K \tag{2}$$

$$\sum_{\rho\in\Omega(a)} x_\rho - U_{e(s)} y_s \leq 0 \qquad s \in S, a \in A_S \tag{3}$$

$$\sum_{s\in S: e(s)=e} n_{e(s)} y_s \leq N_e \qquad e \in E \tag{4}$$

$$x_\rho \in \mathbb{R}^+ \qquad \rho \in \Omega_k,\ k \in K \tag{5}$$

$$y_s \in \{0,1\} \qquad s \in S \tag{6}$$

The objective (1) minimizes cumulative service and cargo transportation cost. As the cargo transportation cost includes the revenue of transporting the cargo this is equivalent to maximizing profit. *The cargo flow constraints* (2) along with non-negativity constraints (5) ensure that all cargo is either transported or forfeited. *The capacity constraints* (3) link the cargo paths with the service capacity installed in the transportation network. *The fleet availability constraints* (4) ensure that the selected services can be operated by the available fleet. Finally, constraints (6) define the service variables as binary.

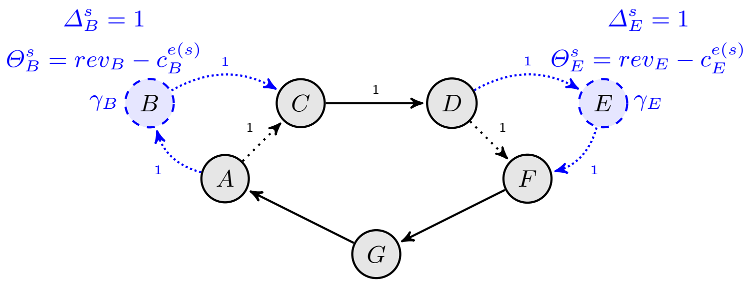
The mathematical model extends the problem description of the LSNDP presented in Brouer et al. (2014a) to handle transit times. The model enforces a weekly frequency resulting in a weekly planning horizon. The path flow formulation of the MCF problem considers transit time restrictions in the definition of a feasible path for a given commodity. Column generation is applied for solving the path flow formulation of the MCF problem, where reduced cost columns are generated by solving a shortest path problem. Introducing transit time restrictions changes the subproblem to a resource constrained shortest path problem and thus the complexity of the subproblem becomes NP-hard. The label setting algorithm from Karsten et al. (2015) is used to solve the cargo routing problem with transit time restrictions during the execution of our algorithm.

In the LSNDP-TT the sailing speed is decisive for the cost of a given service as well as the feasible solution space of the multicommodity flow problem. The majority of all commodities are subject to transshipments and transit time may depend on the choice of speed on multiple services. As a consequence lowering the speed to reduce the cost of a service may make existing cargo routings infeasible due to an increase in transit times. Likewise, increasing speed may result in increased flow in the network as the set of feasible paths increase, but at the same time it will increase the cost of service through the additional fuel burn. The service variables of (1)-(6) are defined for an average speed on all sailings on a round trip and assume a fixed weekly frequency and the resulting speed and cost change from in- or de-creasing by one vessel may be quite significant. However, the proposed algorithm is not optimizing speeds of the individual sailings. The feasible deployment of vessels to maintain weekly frequency will be limited by the minimum and maximum speed.

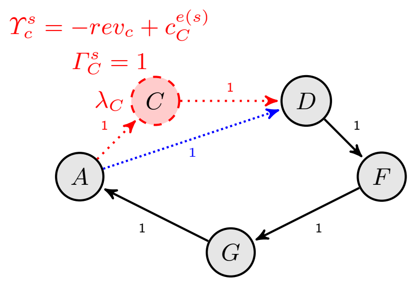
4 Algorithm

The algorithm presented in this paper is an extension of the matheuristic for the LSNDP presented in Brouer et al. (2014b). The algorithm proposed in Brouer et al. (2014b) uses a greedy knapsack based construction heuristic to create an initial set of services, S. Then the core of the matheuristic is executed iteratively to try to improve these using a MIP for each service. The algorithm terminates either when no profitable moves can be found or when a computational time limit is reached. We use the same overall framework in the following and a detailed description and flow chart of the algorithm can be found in Brouer et al. (2014b). The central component in the latter matheuristic is an improvement heuristic, where an integer program is solved as a move operator in a large-scale neighborhood search. The integer program is iteratively solved for a single service using estimation functions for changes in the flow due to insertions and removals of port calls in the service investigated. The solution of the integer program provides a set of moves in the composition of port calls and fleet deployment. Flow changes and the resulting change in the revenue are estimated by solving a series of resource constrained shortest path problems on the residual graph of the current network. This is done for relevant commodities to the insertion/removal of a port call along with an estimation of the change in the vessel related cost with the current fleet deployment.

Given a total estimated change in revenue of rev_i and port call cost of $c_i^{e(s)}$ Figure 1(a) illustrates estimation functions for the change in revenue (Θ_i^s) and duration increase (Δ_i^s) for inserting port i into service s controlled by the binary variable γ_i. The duration controls the number of vessels needed to maintain a weekly frequency of service. Figure 1(b) illustrates the estimation functions for the change in revenue (Υ_i^s) and decrease in duration (Γ_i^s) for removing port i from service s controlled by the binary variable λ_i. Insertions/removals will affect the duration of the service in question and hence the needed fleet deployment modeled by the integer variable ω_s representing the change in the number of vessels deployed.



(a) Blue nodes are evaluated for insertion corresponding to variables γ_i for the set of ports in the neighborhood N^s of service s.



(b) Red nodes are evaluated for removal corresponding to variables λ_i for the set of current port calls F^s on service s.

Fig. 1. Illustration of the estimation functions for insertion and removal of port calls.

4.1 Estimated revenue loss ζ_x due to transit time changes

For considering the transit time in the IP, it is necessary to estimate how insertions and removals of port calls will affect the duration of the existing flow on the service. This means that existing flow must be estimated to have sufficient slack in transit time for the insertions performed or alternatively, existing flow will result in a loss of revenue if it cannot be rerouted within the available transit time on a different path. Figure 2 illustrates a case of a path variable in the current basis of the MCF model, which becomes infeasible due to transit time restrictions when inserting port B on its path.

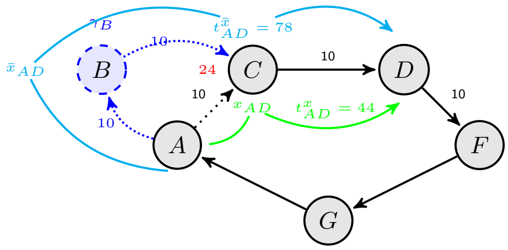


Fig. 2. Insertions/removals affect transit time of the flow. Commodity k_{AD} has a maximum transit time of 48 hours and the insertion of γ_B will make path variable x_{AD} infeasible.

In order to account for the transit time restrictions of the current flow, constraints (14) are added to the IP and a penalty, ζ_x corresponding to losing the cargo, is added to the objective if the transit time slack for an existing path variable becomes negative. This is handled through the variable α_x, where x refers to a path variable with positive flow in the current solution and s_x refers to the current slack time according to the transit time restrictions of the variable. For ease of reading, Table 1 gives an overview of additional sets, constants, and variables used in the IP.

Sets	
N^s	Set of neighbors (potential port call insertions) of s.
X^s	Set of path variables on service s in current flow solution with positive flow.
$N^x \subseteq N^s$	Subset of neighbors with insertion on current path of variable $x \in X^s$.
$F^x \subseteq F^s$	Subset of port calls on current path of variable $x \in X^s$.
L_i	Lock set for port call insertion $i \in N^s$ or port call removal $i \in F^s$.

Constants	
Y_s	Distance of the route associated with s.
K_s	Estimated average speed of the service s.
M_e	Number of undeployed vessels of class e in the current solution.
I_s	Maximum number of insertions allowed in s.
R_s	Maximum number of removals allowed in s.
Δ_i^s	Estimated distance increase if port call $i \in N^s$ is inserted in s.
Γ_i^s	Estimated distance decrease if port call $i \in F^s$ is removed from s.
Θ_i	Estimated profit increase of inserting port call $i \in N^s$ in s.
Υ_i	Estimated profit increase of removing port call $i \in F^s$ from s.
ζ_x	Estimated penalty for cargo lost due to transit time.
s_x	Slack time of path variable x.

Variables	
λ_i	1 if port call $i \in F^s$ is removed from s, 0 otherwise.
γ_i	1 if port call $i \in N^s$ is inserted in s, 0 otherwise.
$\omega_s \in \mathbb{Z}$	Number of vessels added (removed if negative) to s.
α_x	1 if transit time of path variable $x \in X^s$ is violated, 0 otherwise.

Table 1. Overview of sets, constants, and variables used in the IP

Given this notation, the IP is:

$$\max \quad \sum_{i \in N^s} \Theta_i \gamma_i + \sum_{i \in F^s} \Upsilon_i \lambda_i - f_{e(s)} \omega_s - \zeta_x \alpha_x \tag{7}$$

$$\text{s.t.} \quad \frac{Y_s}{K_s} + \sum_{i \in F^s} b_{p(i)} + \sum_{i \in N^s} (\frac{\Delta_i^s}{K_s} + b_{p(i)}) \gamma_i - \sum_{i \in F^s} (\frac{\Gamma_i^s}{K_s} + b_{p(i)}) \lambda_i \leq 24 \cdot 7 \cdot (n_{e(s)} + \omega_s) \tag{8}$$

$$\omega_s \leq M_e \tag{9}$$

$$\sum_{i \in N^s} \gamma_i \leq I_s \tag{10}$$

$$\sum_{i \in F^s} \lambda_i \leq R_s \tag{11}$$

$$\sum_{j \in L_i} \lambda_j \leq |L_i|(1 - \gamma_i) \qquad i \in N^s \tag{12}$$

$$\sum_{j \in L_i} \lambda_j \leq |L_i|(1 - \lambda_i) \qquad i \in F^s \tag{13}$$

$$\sum_{i \in N^x} (\frac{\Delta_i^s}{K_s} + b_{p(i)}) \gamma_i - \sum_{i \in F^x} (\frac{\Gamma_i^s}{K_s} + b_{p(i)}) \lambda_i - UB\alpha_x \leq s_x \qquad x \in X^s \tag{14}$$

$$\lambda_i \in \{0,1\},\ i \in F^s \quad \gamma_i \in \{0,1\},\ i \in N^s \quad \alpha_x \in \{0,1\},\ x \in X^s$$

$$\omega_s \in \mathbb{Z},\ s \in S$$

Category	Instance and description	$\lvert P\rvert$	$\lvert K\rvert$	$\lvert E\rvert$	min v	max v	t.l.
Single-hub	**Baltic** Baltic sea, Bremerhaven as hub	12	22	2	5	7	300
	WAF West Africa, Algeciras as hub	19	38	2	33	51	900
Multi-hub	**Mediterranean** Mediterranean, Algeciras, Tangier, and Gioia Tauro as hubs	39	369	3	15	25	1200
Trade-lane	**Pacific** (Asia-US West)	45	722	4	81	119	3600
	AsiaEurope Europe, Middle East and Far east regions	111	4000	6	140	212	14400
World	**Small** 47 Main ports worldwide identified by Maersk Line	47	1764	6	209	317	10800

Table 2. The instances of the benchmark suite with indication of the number of ports ($|P|$), the number of origin-destination pairs ($|K|$), the number of vessel classes ($|E|$), the minimum (min v) and maximum number of vessels (max v), and the solution time limit in seconds (t.l.). The instances can be found at *http://www.liner-lib.org*

The objective function (7) maximizes the estimated profit increase obtained from removing and inserting port calls, accounting for the estimated change of revenue, transshipment cost, port call cost and fleet cost. As opposed to the IP proposed in Brouer et al. (2014b) the change in revenue may be related to not transporting cargo for which the path duration is estimated to exceed the transit time of the commodity. The number of vessels needed on the service (assuming a weekly frequency) after insertions/removals is estimated in constraint (8) accounting for the change in the service time given the current speed K_s. Constraint (9) ensures that the solution does not exceed the available fleet of vessels. Note that ω_s does not need to be bounded from below by $-n_{e(s)}$ because it is not allowed to remove all port calls. Constraints (10) and (11) limit the number of port call insertions and removals to minimize the error in the computed estimates. The set of port calls affected by an insertion or a removal is fixed by the lock set constraints (12) and (13), respectively. Finally, constraint (14) activates the estimated penalty for lost cargo due to an estimated violation of the transit time for the commodity on this particular path.

5 Computational results

The matheuristic was tested on the benchmark suite *LINER-LIB 2012* described in Brouer et al. (2014a). Table 2 gives an overview of the instances. We have revised the transit time restrictions for a small number of the origin-destination pairs in order to meet critical transit times as our model operates with average sailing speeds. The pairs where the transit times have been revised are those that cannot be satisfied by a direct sailing at 14 knots. The number of revised pairs is 6, 15, 106, and 32 for WAF, Pacific, WorldSmall, and EuropeAsia respectively. They have been revised according to the most recent published liner shipping transit times.

The matheuristic has been coded in `C++` and run on a linux system with an *Intel(R) Xeon(R) X5550* CPU at *2.67GHz* and *24 GB* RAM. The algorithm is set to terminate after the time limits imposed in Brouer et al. (2014a) if the stopping criterion of the embedded simulated annealing procedure is not fulfilled at the time limit.

We fix the berthing time, b_p, to 24 hours for all ports as in Brouer et al. (2014a) and the transshipment time, t_a, is fixed to 48 hours for every connection as the schedule is not considered.

5.1 Computational results for *LINER-LIB 2012*

Table 3 shows that the algorithm can find profitable solutions (negative objective values) for Baltic, WAF, WorldSmall and AsiaEurope. Pacific is unprofitable although both fleet deployment and transport percentage is high. In most instances except the Mediterranean around 85% to 95% of the available cargo is transported on average. At the same time as little as 80% of the fleet in terms of volume is utilized suggesting that further improvements may be achievable as the larger instances all terminate due to the imposed computational time limits.

Instance	**Z(7)**	**D(v)** (%)	**D(|E|)** (%)	**T(v)** (%)	**(S)**
Best - Baltic 1	**-14050**	**100.0**	**100.0**	**87.4**	**101**
Average Baltic	74480	100.0	100.0	86.7	108
Best - WAF 10	$\mathbf{-5.59 \cdot 10^6}$	**83.3**	**85.7**	**97.0**	**255**
Average WAF	$-4.87 \cdot 10^6$	83.3	85.2	94.3	354
Best -Med 5	$\mathbf{2.42 \cdot 10^6}$	**91.9**	**95.0**	**86.9**	**710**
Average Med	$2.70 \cdot 10^6$	90.5	94.0	78.9	737
Best - Pacific 2	$\mathbf{3.05 \cdot 10^6}$	**95.0**	**91.0**	**93.3**	**time**
Average Pacific	$3.65 \cdot 10^6$	94.0	91.9	94.0	time
Best - WorldSmall 5	$\mathbf{-3.54 \cdot 10^7}$	**82.0**	**85.2**	**91.1**	**time**
Average WorldSmall	$-3.15 \cdot 10^7$	82.3	85.4	90.9	time
Best - AsiaEurope 9	$\mathbf{-1.67 \cdot 10^7}$	**84.6**	**90.9**	**88.8**	**time**
Average AsiaEurope	$-1.45 \cdot 10^7$	83.9	91.9	88.5	time

Table 3. Best and Average of 10 runs on an *Intel(R) Xeon(R) X5550* CPU at *2.67GHz* with *24 GB* RAM. Weekly objective value (**Z(7)**); percentage of fleet deployed: as a percentage of the total volume **D(v)**, and as a percentage of the number of ships **D(**$|E|$**)**. **T(v)** is the percentage of total cargo volume transported and (**S**) is the execution time in CPU seconds; time means the solution time limit has been reached.

Table 4 shows that most services operate relatively close to their design speed for the smaller classes, apart from the WorldSmall instances where average

Instance	F450	F800	P1200	P2400	Post P	Super P
Baltic	10.8	13.7				
WAF	11.5	13.2				
Med	11.9	13.7	13.9			
Pacific	12.0	14.2	15.9	18.2		
WorldSmall	12.7	15.5	17.5	19.4	19.4	18.2
AsiaEurope	11.7	13.7	16.5	18.0	19.7	17.6
Design Speed	**12.0**	**14.0**	**18.0**	**16.0**	**16.5**	**17.0**
Max speed	**14.0**	**17.0**	**19.0**	**22.0**	**23.0**	**22.0**

Table 4. Average speed per vessel class over ten runs. Last two rows indicate the design speed and max speed of the corresponding vessel classes. F is Feeder, P is Panamax.

service speed is higher than design speed. The large vessel classes generally have high average speeds. For the WorldSmall and AsiaEurope, we can see in Table 3 that we have excess fleet and by comparing $\mathbf{D(v)}$ and $\mathbf{D}(|E|)$ it can be seen that it is mainly the large vessel classes that are undeployed. This is somewhat surprising as this contradicts the economy of scale of larger vessels. However, Table 4 also shows that the WorldSmall and AsiaEurope operate at very high speeds for the large vessel classes. An explanation could be the fact that we cannot swap vessel classes very well in the algorithm and we are perhaps not able to fill the larger vessels because we have very good utilization on the small services. This needs further investigation.

Table 5 gives statistics on the rejected demand in the solutions. The primary causes are that existing paths do not meet transit time restrictions, that there is no residual capacity or that the OD pair is not connected in the graph. For the smaller instances (Baltic,WAF and Mediterranean) rejection of demand is primarily because the OD pairs are not connected, indicating that it is unprofitable to call these ports. For the larger instances (Pacific, WorldSmall, and AsiaEurope) the demand is primarily rejected due to the transit times that cannot be met (with some variation), and in WorldSmall a significant amount of cargo is also rejected due to lack of capacity. In general comparing the percentage not connected in number of demands (k) compared to the volume (v) not connected indicates that it is the demands with low volume that are not connected. Often these demands are from small feeder ports not visited by the solution because the total volume is very low and it is deemed unprofitable by the algorithm.

6 Conclusions

We have presented a model for the LSNDP-TT introducing transit time restrictions on each individual commodity while maintaining the ability to transship between services. We have extended the matheuristic of Brouer et al. (2014b) to handle transit time restrictions. The core component of the matheuristic is an integer program considering a set of removals and insertions to a service. We

Instance		$\|R\|$	**tt(k)** (%)	**C(k)** (%)	**tt,C(k)** (%)	**L(k)** (%)	**FFE**	**tt(v)** (%)	**C(v)** (%)	**tt,C(v)** (%)	**L(v)** (%)
Baltic	μ	10	0.0	20.8	0.0	79.2	653	0.0	66.4	0.0	33.6
	σ	1	0.0	6.7	0.0	6.7	57	0.0	6.8	0.0	6.8
WAF	μ	7	3.4	16.2	0.0	80.4	489	2.8	28.1	0.0	69.1
	σ	2	7.4	12.5	0.0	9.3	230	6.9	28.4	0.0	26.1
Med	μ	113	32.9	0.7	5.1	61.2	1590	41.9	0.7	7.4	50.0
	σ	25	11.1	0.9	3.1	12.2	521	13.0	1.2	5.1	14.6
Pacific	μ	190	53.4	2.7	15.9	28.0	2629	51.5	10.1	27.6	10.8
	σ	21	11.8	2.3	8.3	5.7	708	17.0	9.5	14.2	4.9
WorldSmall	μ	238	38.3	27.8	23.4	10.5	11635	42.7	24.3	25.9	7.2
	σ	22	11.8	4.9	9.6	13.6	1008	13.5	6.5	10.5	9.5
AsiaEurope	μ	810	37.2	11.7	26.4	24.8	8836	41.6	15.4	31.2	11.9
	σ	66	5.6	5.4	4.0	4.4	871	10.3	6.7	8.9	2.9

Table 5. Statistics on the rejected demand reporting average (μ) and standard deviation (σ) over ten runs. $|R|$ is the number of rejected OD pairs; **tt(k)** is the percentage of OD pairs rejected due only to transit time; **C(k)** is the percentage of OD pairs rejected due only to lack of capacity; **tt,C(k)** is the percentage of OD pairs rejected due to both transit time and lack of capacity; **L(k)** is the percentage of OD pairs not connected; **FFE** is the volume of the rejected demand; **tt(v)** is the percentage of the volume rejected due only to transit time; **C(v)** is the percentage of the volume rejected due only to lack of capacity; **tt,C(v)** is the percentage of volume rejected due to both transit time and lack of capacity; **L(v)** is the percentage of volume rejected because O and D are not connected.

extend the integer program to consider how removals and insertions influence the transit time of the existing cargo flow on the service. Each iteration of the matheuristic provides a set of moves for the current set of services and fleet deployment, which lead to a potential improvement in the overall revenue. The evaluation of the cargo flow for a set of moves requires solving a time constrained multi-commodity flow problem using column generation.

The introduction of transit time constraints changes the estimation functions for the improvement heuristic and the pricing problem of the column generation algorithm from an ordinary shortest path problem to a resource constrained shortest path problem. We apply the specialized label setting algorithm of Karsten et al. (2015) to achieve satisfactory performance.

Extensive computational tests show that it is possible to generate profitable networks for the majority of the instances in *LINER-LIB* and especially for the larger instances the approach generates networks of good quality. However some demand is not served and the fleet is not utilized completely, especially for the larger vessel classes, suggesting that further algorithmic improvements may lead to even better solutions. We expect that especially speed optimization on individual legs as well as more flexibility in terms of possible vessel class swaps could improve the algorithmic performance in terms of profitability for the generated networks as well as the amount of met demand.

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Bibliography

Agarwal, R. and Ergun, O. (2008). Ship scheduling and network design for cargo routing in liner shipping. *Transportation Science*, 42(2):175–196.

Alvarez, J. F. (2012). Mathematical expressions for level of service optimization in liner shipping. *Journal of the Operational Research Society*, 63(6):709–714.

Archetti, C. and Speranza, M. G. (2014). A survey on matheuristics for routing problems. *EURO Journal on Computational Optimization*, 2:223–246.

Brouer, B., Alvarez, J., Plum, C., Pisinger, D., and Sigurd, M. (2014a). A base integer programming model and benchmark suite for liner shipping network design. *Transportation Science*, 48(2):281–312.

Brouer, B., Desaulniers, G., and Pisinger, D. (2014b). A matheuristic for the liner shipping network design problem. *Transportation Research Part E: Logistics and Transportation Review*, 72:42–59.

Christiansen, M., Fagerholt, K., Nygreen, B., and Ronen, D. (2013). Ship routing and scheduling in the new millennium. *European Journal of Operational Research*, 228(3):467–483.

Gelareh, S., Nickel, S., and Pisinger, D. (2010). Liner shipping hub network design in a competitive environment. *Transportation Research Part E: Logistics and Transportation Review*, 46(6):991–1004.

Karsten, C. V., Pisinger, D., Ropke, S., and Brouer, B. D. (2015). The time constrained multi-commodity network flow problem and its application to liner shipping network design. *Transportation Research Part E: Logistics and Transportation Review*, 76:122–138.

Liu, Z., Meng, Q., Wang, S., and Sun, Z. (2014). Global intermodal liner shipping network design. *Transportation Research Part E: Logistics and Transportation Review*, 61:28–39.

Meng, Q., Wang, S., Andersson, H., and Thun, K. (2014). Containership routing and scheduling in liner shipping: Overview and future research directions. *Transportation Science*, 48(2):265–280.

Mulder, J. and Dekker, R. (2014). Methods for strategic liner shipping network design. *European Journal of Operational Research*, 235(2):367–377.

Plum, C., Pisinger, D., and Sigurd, M. M. (2014). A service flow model for the liner shipping network design problem. *European Journal of Operational Research*, 235(2):378–386.

Reinhardt, L. B. and Pisinger, D. (2012). A branch and cut algorithm for the container shipping network design problem. *Flexible Services and Manufacturing Journal*, 24(3):349–374.

Wang, S. and Meng, Q. (2014). Liner shipping network design with deadlines. *Computers and Operations Research*, 41(1):140–149.

Wang, S., Meng, Q., and Sun, Z. (2013). Container routing in liner shipping. *Transportation Research Part E: Logistics and Transportation Review*, 49(1):1–7.